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TECHNICAL REPORT No. 10

COMPRESSIBLE FLOW THROUGH REED VALVES FOR PULSE JET ENGINES, II. CLAMPED REED VALVES

by

Paul Torda

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INTRODUCTION

In a previous report, Ref. 1, the a.. inflow through automatically operating hinged reed valves was treated. This paper deals with the inflow analysis between clamped reed valves used on conventional pulse jet engines. For a detailed discussion of background reference should be made to Ref. 1.

Again, as in Ref. 1, the basic postulate is made that the reeds form smooth nozzles during their motion, thereby increasing the inflow efficiency. Since the bending stresses are reduced by eliminating oscillations about the momentarily bent shapes of the reeds, the reed endurance will also be increased.

This analysis, as the one Ref. 1, was based on the theory of non-steady, compressible, non-viscous flow with isentropic change of state of the gas, and employs a quasi-one-dimensional approach. The resulting non-linear differe..tial equations have been integrated in closed form. To solve the problem in a general manner, the time variation of the flow velocity at an arbitrary time and arbitrary cross section of the reed nozzle was prescribed and the corresponding pressure distribution on the combustion chamber side of the reeds was calculated. The inverse method would allow particular numerical solutions only. The prescribed boundary and transition conditions have been satisfied. Although the exact transition conditions between the inflow and the aero-thermodynamic proc' ss of the engine were not known because of lack of experimental as well as theoretical evidence, it is thought that a sufficiently broad range of such conditions is covered in the analysis as to include all experimentally or theoretically determined transition conditions likely to arise in future investigations.

AIR INFLOW ANALYSIS

In this analysis the flow upstream of the valves has been assumed to be parallel to the valve center planes, i.e. a short cowl of large diameter was assumed ahead of the valve bank, which is built up of a large number of individual valves.

The quasi one-dimensional approach used takes into account the time and space variation of area, flow pressure, and flow velocity between the reeds. The analysis considers non-steady, compressible, non-viscous flow between clamped reed valves with isentropic change of state during the period of opening.

By satisfying the Euler dynamic equations, the continuity equation, and the equation of change of state, and by prescribing that the momentary nozzle shapes be smooth, the flow variables between a pair of reeds were determined. The interaction between the inflow conditions, the valves, and the combustion chamber pressure is taken into account by additionally satisfying the equation of forced reed vibration. Thus, the pressure variation on the combustion chamber side of the reeds is calculated as a result of the analysis. The use of the results for design purposes is discussed in the beginning of section (C).

The basic equations and part of their integration are identical to those for the *hinged* valves, Ref. 1. Thus only the basic equations and the main steps of integration rather than all the intermediate steps are given in this analysis.

A complete list of symbols is included at the end of the report.

(A) NOTATION AND BASIC EQUATIONS DEFINING THE FLOW

The following notation is used in this analysis, see Figure 1:

$A = A(x,t)$	cross sectional area between a pair of reeds	(a.1)
$\rho = \rho(x,t)$	density of gas	
$p = p(x,t)$	pressure of the gas	
$u = u(x,t)$	velocity of the flow in X direction	
x	space variable	
t	time variable	
$\gamma = \dfrac{c_p}{c_v}$	adiabatic constant	
c_p and c_v	specific heat of the gas at constant pressure and constant volume respectively.	
subscript o	free stream conditions	

The equation of continuity is

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho A u)}{\partial x} = 0 \qquad \text{(a.2)}$$

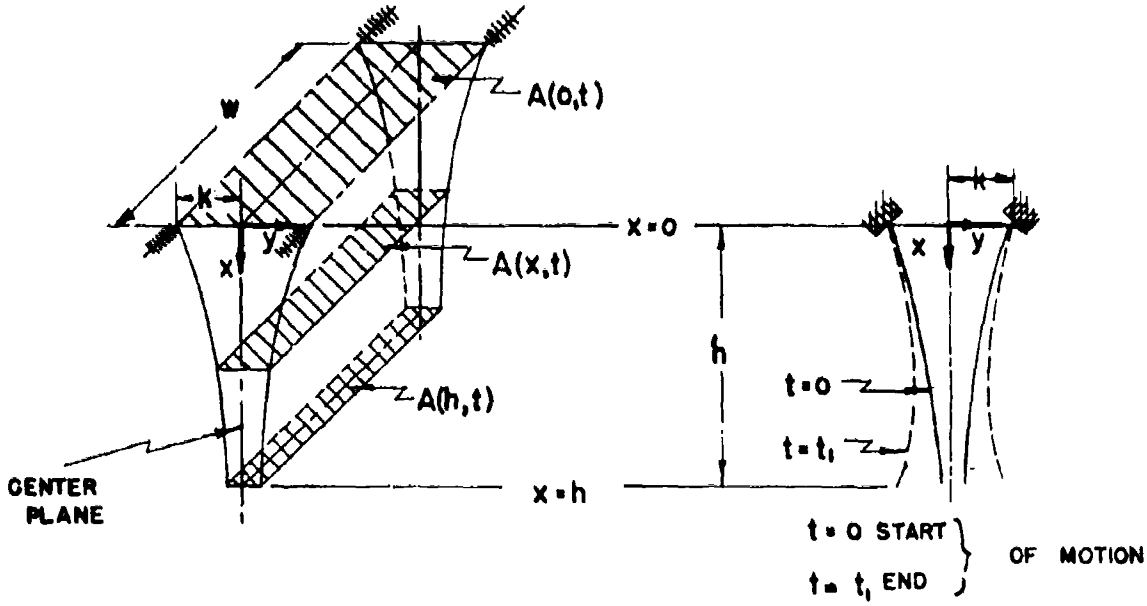

FIGURE 1

The Euler dynamic equation of motion is

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \qquad \text{(a.3)}$$

The equation of isentropic change of state is

$$\frac{p}{p_o} = \left(\frac{\rho}{\rho_o}\right)^{\gamma} \qquad \text{(a.4)}$$

(B) INTEGRATION OF THE BASIC EQUATIONS

A new variable, the product of density and cross-sectional area, is introduced as

$$B = B(x,t) = \rho(x,t)\ A(x,t) \qquad \text{(b.1)}$$

and the continuity equation (a.2) can then be integrated to yield

$$u = \frac{1}{B}\left[-\int\frac{\partial B}{\partial t}dx + K_1(t)\right] \qquad \text{(b.2)}$$

where $K_1(t)$ is an arbitrary function of time arising in the integration.

It is assumed that in the solution of equation (b.1) the variables are separable, say

$$B(x,t) = X(x)\ T(t) = X \cdot T \qquad \text{(b.3)}$$

then

$$X(x) = be^{cx} \tag{b.4}$$

satisfies all the equations and yields desirable valve shapes.

Expressing (b.2) in terms of (b.3) and (b.4), yields

$$u = \frac{1}{bT} e^{-cx} K_1(t) - \frac{1}{cT}\frac{dT}{dt} \tag{b.5}$$

Denoting

$$c_1 = \frac{p_o \gamma}{\rho_o^{\gamma}} \tag{b.6}$$

expressing $\partial p/\partial x$ from equation (a.4), using (b.5) for u and its derivatives, and then substituting these values into the Euler dynamic equation (a.3) and integrating, the following expression is found for the density

$$\rho(x,t) = \left\{ \frac{\gamma-1}{c_1} \left[\frac{x}{c} \left\{ \frac{1}{T}\frac{d^2T}{dt^2} - \frac{1}{T^2}\left(\frac{dT}{dt}\right)^2 \right\} - \frac{e^{-2cx}}{2b^2T^2}\Big(K_1(t)\Big)^2 + \frac{e^{-cx}}{bcT}\frac{dK_1(t)}{dt} + K_2(t) \right] \right\}^{\frac{1}{\gamma-1}}$$

where $K_2(t)$ is an arbitrary function of time arising in the integration. (b.7)

Inserting (b.7) in (b.3), the expression for the area variable can be written as

$$A(x,t) = \frac{B}{\rho} = be^{cx}\, T \left\{ \frac{\gamma-1}{c_1} \left[\frac{x}{c} \left\{ \frac{1}{T}\frac{d^2T}{dt^2} - \frac{1}{T^2}\left(\frac{dT}{dt}\right)^2 \right\} - \frac{e^{-2cx}}{2b^2T^2}(K_1(t))^2 + \frac{e^{-cx}}{bcT}\frac{dK_1(t)}{dt} + K_2(t) \right] \right\}^{\frac{1}{1-\gamma}} \tag{b.8}$$

The expressions for velocity (b.5), density (b.7), and area distribution (b.8) are general solutions of the basic equations. They contain arbitrary constants and arbitrary functions of time which have to be determined from the conditions prescribed by the particular problem.

(C) SOLUTION OF THE BOUNDARY VALUE PROBLEM

The function $T(t)$ describing the time history of inflow and valve opening is as yet undetermined. The transition conditions between the inflow and the aero-thermodynamic process of the combustion chamber should be used to determine this function. This was not possible since the transition conditions involved were unknown. The determination of $T(t)$ was effected thus:

If it is assumed that the velocity function at the entrance, $u(0,t)$, is known by some means then the function $T(t)$ can be evaluated. By prescribing such functions $u(0,t) = f(t)$ of various nature, $T(t)$ was determined. The velocity, density, and pressure in the flow, and the area distribution of the reed valves were calculated for each selected function $u(o,t) = f(t)$. The examples included are insufficient for design use. Towards this end, many more examples should be calculated by the method given in this paper. It would then be possible to select the proper valve parameters for any particular design problem by comparing the results of such examples with the combustion chamber pressure variation of interest.

The initial and boundary conditions of the present problem are assumed as follows:

1. The area between a pair of reeds must be a constant at the entrance ($x=o$) for any time (t).
2. The reeds are clamped, i.e., the tangent of the reeds at $x=0$ must be a constant during the whole valve motion.
3. The appropriate velocity function, selected for the evaluation of $T(t)$, must be prescribed.
4. At the start of the inflow ($t=0$), the density at the inlet section ($x=0$) was assumed to be the stagnation density (ρ_S), and the inlet area constant.
5. The start of the inflow ($t=o$) was chosen at a time when the reeds already are open slightly. This allowed the use of an exponential function for the space function $X(x)$ of the analysis. Should the starting time be chosen when the valves are fully closed, an additive space function would have to be used in order that the area be zero at $x=h$. The use of such an additive function would only complicate the already lengthy numerical work and would not yield significant results, since at the beginning of the reed movement the flow is well behaved. Therefore, the additive space function was left out and only the exponential term of the space function used. However, the time interval (Δt) during which the valves open slightly together with the amount of the slight opening $[A(h,o)]$ can be made arbitrarily small without adverse effects on the results.

   At the start the density distribution along x should be nearly constant and even at $x=h$ should be equal approximately to the stagnation density (ρ_S).
6. At the end of the opening ($t=t_1$), the density at the exit section, $\rho(h,t_1)$, and the exit area, $A(h,t_1)$, should have prescribed values.

The above conditions can be stated mathematically as:

$$\left.\begin{array}{ll}
1. & A(0,t) = A_{o,t} = A_{o,o} \\
2. & \left(\dfrac{\partial y}{\partial x}\right)_{x=o} = \eta = \text{const.} \\
3. & u(0,t) = f(t) \\
4. & B(0,0) = \rho_S A_{o,o} = \rho_S A_{o,t} \\
5. & B(h,0) \cong \rho_S A_{h,o} \\
6. & B(h,t_1) = \mu \rho_S \sigma A_{o,o}
\end{array}\right\} \quad (c.1)$$

where

$A_{o,o} = A_{o,t} = 2kw$ = area at $x = 0$

$A_{h,o}$ = area at $x=h$, $t=0$

ρ_s = stagnation density

μ, σ = constants

$f(t)$ = given function of time

t_1 = time needed for opening of the valves

η = tangent of the reed at built in end.

Using the boundary value (c1.1), expression (b.8) for the area distribution allows the evaluation of the arbitrary time function $K_1(t)$.

$$K_1(t) = \frac{c_1}{\gamma-1}\left[\frac{A_{o,t}}{bT}\right]^{(1-\gamma)} + \frac{[K_1(t)]^2}{2b^2T^2} - \frac{dK_1(t)}{dt}\frac{1}{bcT} \tag{c.2}$$

The boundary value (c1.2), together with the derivative of the area (b.3) with respect to x, yields the following differential equation in $T(t)$ and $K_1(t)$

$$\left.\begin{aligned} \alpha &= -T^{(1-\gamma)}\left[\frac{1}{c}\frac{d}{dt}\left(\frac{dT}{Tdt}\right) + \frac{c[K_1(t)]^2}{b^2T^2} - \frac{dK_1(t)}{bTdt}\right] \\ \text{where}\quad \alpha &= \left[\eta - \frac{cA_{o,t}}{2w}\right]\frac{2wc_1}{b}\left(\frac{b}{A_{o,t}}\right)^{\gamma} \end{aligned}\right\} \tag{c.3}$$

Condition (c1.3) used in conjunction with equation (c.3) yields a linear differential equation in a new variable

$$v = T^{(1-\gamma)} \tag{c.4}$$

in the following manner:

The velocity function (b.5) for x=0 yields

$$u(o,t) = \frac{K_1(t)}{bT} - \frac{1}{cT}\frac{dT}{dt} = f(t) \tag{c.5}$$

from which

$$K_1(t) = bT\left[f(t) + \frac{1}{cT}\frac{dT}{dt}\right] \tag{c.6}$$

Substituting (c.6) into equation (c.3) and introducing the new variable, v, as given by (c.4), the linear first order differential equation resulting for v is

$$\frac{dv}{dt} + v\left[(1-\gamma)\left\{cf(t) - \frac{df(t)}{fdt}\right\}\right] + \frac{(1-\gamma)\alpha}{f} = 0 \qquad (c.7)$$

and its solution

$$v(t) = e^{-\int \left\{(1-\gamma)\left[cf(t) - \frac{df(t)}{fdt}\right]\right\}dt} \times \left[\vphantom{\int}\right. \qquad (c.8)$$
$$\left. \times\left[\int\left\{-\frac{(1-\gamma)\alpha}{f}\, e^{\int \left\{(1-\gamma)\left[cf(t) - \frac{df(t)}{fdt}\right]\right\} dt}\, dt\right\} + \beta\right]\right.$$

where β is an arbitrary constant arising in the integration.

Once $v(t)$ has been evaluated, the whole time history of the inflow and reed movement is determined, since $T(t)$, and thus $A(x,t)$, $\rho(x,t)$, $p(x,t)$, and $u(x,t)$, can be expressed.

The choice of $f(t)$ determines whether $v(t)$ can be expressed in a closed form. In such cases where the desired $u(o,t)=f(t)$ does not lend itself to a closed form solution for $v(t)$, the computations must be made using numerical methods, e.g. graphical integration.

Four different functions $f(t)$ were selected as examples and the special solutions presented.

(D) SPECIAL SOLUTIONS

The four functions $u(o,t)=f(t)$, for which calculations are presented, were prescribed as

$$\left.\begin{aligned}
&\textbf{Case I} - u(o,t) = f(t) = \frac{m}{n+t}\\
&\textbf{Case II} - u(o,t) = f(t) = mte^{nt} + m_o\\
&\textbf{Case III} - u(o,t) = f(t) = \frac{m}{c-e^{m(t+n)}}\\
&\textbf{Case IV} - u(o,t) = f(t) = m_o = \text{const.}
\end{aligned}\right\} \quad (d.1)$$

In the following detailed analysis for Case I is given. For the other cases the analysis is similar, therefore, only the final results are included.

Case I. Expression (c.8) gives the general solution of the differential equation for $T^{(1-\gamma)} = v$ in terms of $f(t)$. Using (d.1) for this case, or

$$u(o,t) = f(t) = \frac{m}{n+t} \qquad (d.1)$$

and evaluating the integrals, v(t) can be calculated.

With

$$\int(1-\gamma)[cf(t) - \frac{df(t)}{fdt}]dt = (1-\gamma)\int[c\frac{m}{n+t} - \frac{df(t)}{fdt}]dt =$$

$$= log_e(n+t)^{mc(1-\gamma)} + log_e(n+t)^{(1-\gamma)} = log_e(n+t)^{(1-\gamma)(mc+1)}$$

$$v(t) = -(1-\gamma)\,\frac{\alpha}{m}\,\frac{(n+t)^2}{(1-\gamma)(cm+1)+2} + \beta(n+t)^{(\gamma-1)(cm+1)} \qquad \text{(d.2)}$$

Then, from (c.4)

$$T(t) = \left\{ \frac{\alpha(\gamma-1)(n+t)^2}{m[(1-\gamma)(cm+1)+2]} + \beta(n+t)^{(\gamma-1)(cm+1)} \right\}^{\frac{1}{1-\gamma}} \qquad \text{(d.3)}$$

Using (d.3) for $T(t)$ and its de ivatives and substituting them into (c.6), the arbitrary time function, $K_1(t)$, as determined from condition (c1.3), becomes

$$K_1(t) = \frac{b}{c}\left\{v(t)\right\}^{\frac{1}{1-\gamma}}\Bigg[\frac{cm}{n+t} + \frac{1}{1-\gamma}\left\{v(t)\right\}^{-1}\Bigg\{\frac{2\alpha(\gamma-1)(n+t)}{m[(1-\gamma)(cm+1)+2]} +$$

$$+ \beta(\gamma-1)(cm+1)(n+t)^{[(\gamma-1)(cm+1)-1]}\Bigg\}\Bigg] \qquad \text{(d.4)}$$

where $v(t)$ is given by (d.2).

With this value of $K_1(t)$, the area expression (b.8) becomes

$$A(x,t) = be^{cx}[v(t)]^{\frac{1}{1-\gamma}}[\rho(x,t)]^{-1} \qquad \text{(d.5)}$$

and the expression for the density

$$\rho(x,t) = \Bigg\{ \frac{\gamma-1}{c_1}\Bigg[[v(t)]^{-1}\Big\{\frac{2\alpha}{m[(1-\gamma)(cm+1)+2]} + \beta(cm+1)[(\gamma-1)(cm+1) -$$

$$-1](n+t)^{[(\gamma-1)(cm+1)-2]}\Big\} \frac{1-cx-e^{-cx}}{c^2} +$$

$$+[v(t)]^{-1}\Big\{\frac{2\alpha}{m[(1-\gamma)(cm+1)+2]} + \beta(cm+1)(n+t)^{[(\gamma-1)(cm+1)-2]}\Big\} \frac{m(e^{-2cx}-e^{-cx})}{c} +$$

$$+[v(t)]^{-2}\Big\{\frac{2\alpha}{m[(1-\gamma)(cm+1)+2]} + \beta(cm+1)(n+t)^{[(\gamma-1)(cm+1)-2]}\Big\}^2 \cdot \Big\{$$

$$\cdot \Big\{\frac{(n+t)^2[2\gamma(e^{-cx}-1)-2c(1-\gamma)x+1-e^{-2cx}}{2c^2}\Big\} +$$

$$+ \frac{m[cm(1-e^{-2cx})+2(1-e^{-cx})]}{2c(n+t)^2} + \frac{c_1}{\gamma-1}\left(\frac{A_{o,t}}{b}\right)^{(1-\gamma)} [v(t)]^{-1}\Bigg]\Bigg\}^{\frac{1}{\gamma-1}} \qquad (d.6)$$

The velocity function can now be written

$$u(x,t) = \frac{m}{n+t}e^{-cx} + \frac{n+t}{c}[v(t)]^{-1} \cdot \Bigg[\qquad (d.7)$$

$$\cdot\left[\frac{2\alpha}{m[(1-\gamma)(cm+1)+2]} + \beta(cm+1)(n+t)^{[(\gamma-1)(cm+1)-2]}\right](1-e^{-cx})$$

In the above expressions, $v(t)$ is given by (d.2).

From boundary conditions (c 1.4), i.e.

$$B(o,o) = A_{o,t}\rho_s \qquad (c1.4)$$

$$A_{o,t}\rho_s = b\Big\{\frac{\alpha(\gamma-1)\, n^2}{m[(1-\gamma)(cm+1)+2]} + \beta n^{[(\gamma-1)(cm+1)]}\Big\}^{\frac{1}{1-\gamma}} \qquad (d.8)$$

and from boundary conditions (c1.5), i.e.

$$B(h,o) = A_{h,o}\rho_s \qquad (c1.5)$$

$$A_{h,o}\rho_s = be^{ch}\Big\{\frac{\alpha(\gamma-1)\, n^2}{m[(1-\gamma)(cm+1)+2]} + \beta n^{[(\gamma-1)(cm+1)]}\Big\}^{\frac{1}{1-\gamma}} \qquad (d.9)$$

From (d.8) and (d.9) the values of c and of β can be calculated to

$$c = \frac{1}{h}\, log_e \left(\frac{A_{h,c}}{A_{o,c}} \right) \tag{d.10}$$

$$\beta = \left\{ \left(\frac{A_{o,t}\rho_s}{b} \right)^{(1-\gamma)} - \frac{n^2(\gamma-1)\alpha}{m[(1-\gamma)(cm+1)+2]} \right\} n^{[(1-\gamma)(cm+1)]} \tag{d.11}$$

Finally, from boundary condition (c1.6), i.e.

$$B(h,t_1) = \mu A_{o,t}\sigma\rho_s \tag{c1.6}$$

where $\mu < 1$ and $\sigma < 1$

the value of η can be determined as

$$\left(\frac{\partial y}{\partial x} \right)_{x=o} = \eta = \frac{A_{o,t}}{2\omega} \left\{ \rho_s{}^{(1-\gamma)} \left[\left(\mu\sigma \frac{A_{o,t}}{A_{h,o}} \right)^{(1-\gamma)} - \left(\frac{n+t_1}{n} \right)^{(\gamma-1)(cm+1)} \right] \cdot \right.$$

$$\left. \cdot \frac{m[(1-\gamma)(cm+1)+2]}{c_1(\gamma-1)[(n+t_1)^2-n^2 \left(\frac{n+t_1}{n} \right)^{(\gamma-1)(cm+1)}]} + c \right\} \tag{d.12}$$

The final expressions for area, density, and velocity distribution to be used in the numerical calculations are

$$A(x,t) = A_{o,t}\rho_s\, e^{-cx}\, M^{\frac{1}{1-\gamma}}[\rho]^{-1} \tag{d.13}$$

$$\rho(x,t) = \left\{ \frac{\gamma-1}{c_1} \left[\frac{1-cx-e^{-cx}}{c^2n^2} M^{-1} \left\{ \frac{2N}{\gamma-1} + [(\gamma-1)(cm+1)-1]Q \right\} + \right. \right.$$

$$+ \frac{m}{cn^2}(e^{-2cx}-e^{-cx})\, M^{-1} \left[\frac{2N}{\gamma-1} + Q \right] + \tag{d.14}$$

$$+ \frac{2\gamma(e^{-cx}-1)-2c(1-\gamma)x+(1-e^{-2cx})}{2c^2n^2} \left(\frac{n+t}{n} \right)^2 M^{-2} \left[\frac{2N}{\gamma-1} + Q \right]^2 +$$

$$\left. \left. + [cm(1-e^{-2cx}) + 2(1-e^{-cx})] \frac{m}{2c(n+t)^2} + \frac{c_1}{\gamma-1} \rho_s{}^{(\gamma-1)} M^{-1} \right] \right\}^{\frac{1}{\gamma-1}}$$

where

$$\left.\begin{aligned}
N &= \frac{(\mu\sigma e^{-ch})^{(1-\gamma)} - \left(\frac{n+t_1}{n}\right)^{(\gamma-1)(cm+1)}}{\left(\frac{n+t_1}{n}\right)^{2} - \left(\frac{n+t_1}{n}\right)^{(\gamma-1)(cm+1)}} \\
M &= N\left(\frac{n+t}{n}\right)^{2} + \left(\frac{n+t}{n}\right)^{(\gamma-1)(cm+1)}(1-N) \\
Q &= (1-N)(cm+1)\left(\frac{n+t}{n}\right)^{[(\gamma-1)(cm+1)-2]}
\end{aligned}\right\} \quad (d.15)$$

$$u(x,t) = \left(\frac{m}{n+t}\right)e^{-cx} + \frac{n+t}{cn^2}\left\{N\left(\frac{n+t}{n}\right)^2 + (1-N)\left(\frac{n+t}{n}\right)^{(\gamma-1)(cm+1)}\right\}^{-1} \cdot \left\{\frac{2N}{\gamma-1} + (1-N)(cm+1)\left(\frac{n+t}{n}\right)^{(\gamma-1)(cm+1)}\right\}(1-e^{-cx}) \quad (d.16)$$

Case II. Using expression (d.1) for this case, or

$$u(o,t) = f(t) = mte^{nt} + m_o \quad (d.1)$$

the resulting expressions for area, density, and velocity distribution to be used in the numerical calculations are:

$$A(x,t) = A_{o,t}\rho_s\ e^{cx}\ fe^{-c\lambda_1}M^{\frac{1}{1-\gamma}}[\rho]^{-1} \quad (d.17)$$

$$\begin{aligned}
\rho(x,t) = \Bigg\{ & \frac{\gamma-1}{c_1}\left[(cx+e^{-cx}-1)\left[\frac{mne^{nt}(nt+2)}{fc^2} - \frac{f'}{c}\right] + \right. \\
& + \frac{1}{2}(1-e^{-2cx}-2cx)\left(\frac{f'}{cf}\right)^2 + [2(\gamma-1)cx+1-e^{-2cx}+2\gamma(e^{-cx}-1)] \cdot \\
& \cdot \frac{f^{2(\gamma-2)}e^{2(1-\gamma)c\lambda_1}\lambda_2^2}{2c^2(\gamma-1)^2[N]^2}M^{-2} + \\
& + [(2-\gamma)cx-1+e^{-2cx}-\gamma(e^{-cx}-1)]\frac{f^{(\gamma-3)}\ f'e^{(1-\gamma)c\lambda_1}\lambda_2}{c^2(\gamma-1)N}M^{-1} + \\
& \left. + \left[(\gamma-1)cx+\gamma(e^{-cx}-1) + \frac{cc_1\rho_s{}^{(\gamma-1)}N}{\lambda_2}\right]\frac{f^{(\gamma-1)}e^{(1-\gamma)c\lambda_1}\lambda_2}{c(\gamma-1)N}M^{-1}\right]\Bigg\}^{\frac{1}{\gamma-1}}
\end{aligned} \quad (d.18)$$

where

$$\left.\begin{aligned}
f &= mte^{nt} + m_o \\
f' &= me^{nt}(nt+1) \\
\lambda_1 &= \int f dt = \frac{m}{n^2}e^{nt}(nt-1) + m_o t \\
\lambda_2 &= \left[\frac{\mu_o e^{c[\lambda_1 t_1 - h]}}{[f]_{t_1}}\right]^{(1-\gamma)} - \left\{\frac{e^{\frac{-cm}{n^2}}}{m_o}\right\}^{(1-\gamma)} \\
N &= \int_o^{t_1} f^{(\gamma-2)} e^{(1-\gamma)c\lambda_1} dt \\
M &= \lambda_2 \frac{\int_o^t f^{(\gamma-2)} e^{(1-\gamma)c\lambda_1} dt}{N} + \left\{\frac{e^{-\frac{cm}{n^2}}}{m_o}\right\}^{(1-\gamma)}
\end{aligned}\right\} \quad (d.19)$$

$$u(x,t) = \frac{1}{c}\left[\left\{\frac{f'}{f} - \frac{f^{(\gamma-2)}e^{(1-\gamma)c\lambda_1}\lambda_2}{(\gamma-1)N} M^{-1}\right\}(e^{-cx}-1) + cf\right] \quad (d.20)$$

Case III. Using expression (d.1) for this case, or

$$u(o,t) = f(t) = \frac{m}{c-e^{m(t+n)}} \quad (d.1)$$

the resulting expressions for area, density, and velocity distribution to be used in the numerical calculations are:

$$A(x,t) = A_{o,t}\rho_s \, e^{cx} \, N^{\frac{1}{1-\gamma}} [\rho]^{-1} \quad (d.21)$$

$$\rho(x,t) = \left\{\frac{\gamma-1}{c_1}\left[\frac{m^2}{2c^2}[\lambda_1-\lambda_2]^2 N^{-2} [2c(\gamma-1)x + (1-e^{-2cx}) + 2\gamma(e^{-cx}-1)] + \right.\right.$$

$$+ \frac{m}{c}f[\lambda_1-\lambda_2] N^{-1}[e^{-cx}-e^{-2cx}] + \frac{m^2}{c^2}[\lambda_1-(\gamma-1)\lambda_2]N^{-1}[cx + e^{-cx} - 1] +$$

$$\left.\left. + \frac{f^2}{2c}[c(1-e^{-2cx}) + 2(e^{-cx}-1)e^{m(t+n)}] + \frac{c_1}{\gamma-1}\rho_s^{(\gamma-1)}N^{-1}\right]\right\}^{\frac{1}{\gamma-1}} \quad (d.22)$$

where

$$\left.\begin{aligned}
\lambda_1 &= \frac{M}{2-\gamma}\, e^{m(t+n)} \\
\lambda_2 &= e^{(\gamma-1)mt}\left[1+M\left\{ c+\frac{\gamma-1}{2-\gamma}\, e^{mn}\right\}\right] \\
N &= \lambda_2 - M\left[c+\frac{\gamma-1}{2-\gamma}\, e^{m(t+n)}\right] \\
f &= \frac{m}{c-e^{m(t+n)}} \\
M &= \frac{(\mu\sigma e^{-ch})^{(1-\gamma)} - e^{(\gamma-1)mt_1}}{e^{(\gamma-1)mt_1}\left[c+\frac{\gamma-1}{2-\gamma}\, e^{mn}\right] - c - \frac{\gamma-1}{2-\gamma}\, e^{m(t_1+n)}}
\end{aligned}\right\} \tag{d.23}$$

$$u(x,t) = \frac{1}{c}\left[cfe^{-cx} + \frac{m(\lambda_1-\lambda_2)}{N}\,(e^{-cx}-1)\right] \tag{d.24}$$

Case IV. Using expression (d.1) for this case, or

$$u(o,t) = f(t) = m_o = \text{const.} \tag{d.1}$$

the resulting expressions for area, density, and velocity distribution to be used in the numerical calculations are:

$$A(x,t) = \varepsilon A_{o,t}\rho_S\, e^{cx}\, M^{\frac{1}{1-\gamma}}[\rho]^{-1} \tag{d.25}$$

$$\begin{aligned}
\rho(x,t) = \Bigg\{ \frac{\gamma-1}{c_1}\Bigg[& m_o^2\, NM^{-1}[(1-\gamma)cx+e^{-2cx}-1-\gamma(e^{-cx}-1)] + \\
& + m_o^2 N^2 M^{-2}[(\gamma-1)\,cx - \tfrac{1}{2}\,(e^{-2x}-1) + \gamma(e^{-cx}-1)] - \\
& - \frac{m_o^2}{2}\,(e^{-2cx}-1) + \frac{c_1}{\gamma-1}(\varepsilon\rho_S)^{(\gamma-1)}M^{-1}\Bigg]\Bigg\}^{\frac{1}{\gamma-1}}
\end{aligned} \tag{d.26}$$

where

$$N = \frac{e^{(\gamma-1)cm_0 t}\left[\left(\frac{\mu\sigma}{\varepsilon}\, e^{-ch}\right)^{(1-\gamma)} - 1\right]}{e^{(\gamma-1)cm_0 t_1} - 1},$$

$$M = \frac{\left(\frac{\mu\sigma}{\varepsilon}\, e^{-ch}\right)^{(1-\gamma)}\left(e^{(\gamma-1)cm_0 t} - 1\right) + e^{(\gamma-1)cm_0 t_1} - e^{(\gamma-1)(cm_0 t)}}{e^{(\gamma-1)cm_0 t_1} - 1} \tag{d.27}$$

$$u(x,t) = m_0 e^{-cx} + m_0\, NM^{-1}\,(1 - e^{-cx}) \tag{d.28}$$

In this case two of the boundary conditions had to be modified as follows:

at $x = o$ and $t = o$ $A = A_{o,t}$ and $\rho = \varepsilon\rho_s$

and thus $B(o,o) = \varepsilon A_{o,t}\rho_s$

at $x = h$ and $t = o$ $A = A_{h,o}$ and $\rho = \lambda\rho_s$

and thus $B(h,o) = \lambda A_{h,o}\rho_s$

and the constant c becomes

$$c = \frac{1}{h}\, \log_e\left(\frac{\lambda A_{h,o}}{\varepsilon A_{o,t}}\right)$$

(E) MASS DISTRIBUTION OF REEDS AND COMBUSTION CHAMBER PRESSURE VARIATION

In the beginning of *Air Inflow Analysis* it was stated that the pressure variation on the combustion chamber side of the reeds is calculated as a result of the analysis by additionally satisfying the equation of forced reed vibration:

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right) + m^*\frac{\partial^2 y}{\partial t^2} = q(x,t) \tag{e.1}$$

where

$y = \frac{A}{2w}$

w = depth of reeds

m^* = mass of reeds per unit length

$q(x,t)$ = forcing function, the difference of the pressure forces on both sides of the reeds.

(e.2)

For any prescribed mass distribution of the reeds used in the numerical calculations, the pressure variation on the combustion chamber side of the reeds can be determined as

$$q_c(x,t) = q_i(x,t) - \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right) - m^*\frac{\partial^2 y}{\partial t^2} \qquad \text{(e.3)}$$

where

$q_c(x,t)$ = pressure on combustion chamber side of reeds (per unit length).
$q_i(x,t)$ = pressure on inflow side of reeds (per unit length).

(F) REMARKS ON NUMERICAL EXAMPLES

No numerical examples are included in this report similar to those given in Ref. 1 due to the lack of time and personnel.

Judging from the experience gained during the work done on numerical calculations for the hinged valves, Ref. 1, and from the calculations made to date on the clamped valves, there should be little if any difference in valve shapes, movement, velocity, pressure and mass distribution between the clamped and hinged reed valves. Inspection of the valve motion curves for the hinged reed valves, included in Ref. 1, shows virtually no change of the tangent to the reed at the entrance section during the total movement. Thus, the main valve motion is due to bending of the reeds during opening.

The expressions for the present case, though necessarily more involved, should result in similar curves.

It should be of interest to the designer to have numerical examples worked out for the clamped reed valves. Such examples would facilitate the choice of valve design most suitable for the particular problem of interest. No great difficulty is expected for trained personnel in doing such numerical computations as suggested in this paper. For the actual procedure the method used in Ref. 1 should be followed.

CONCLUSIONS

Numerical results which may be obtained by means of the above analysis and those of Ref. 1 involve laborious computations. To overcome this disadvantage, a method of approximation more suitable for engineering applications has been worked out together with numerical examples. These results will be reported in the near future.

REFERENCES

1. Torda, Paul; Villalba, I.P.; and Brick, J.H.: *Compressible Flow Through Reed Valves For Pulse Jet Engines. I. Hinged Reed Valves*, Technical Report No. 9, Project Squid, 14 June 1948.

LIST OF SYMBOLS

$A = A(x,t)$	cross sectional area between a pair of reeds.
$A_{o,o} = A_{o,t}$	cross sectional area at $x = o$.
$A_{h,o}$	cross sectional area at $x = h$ and $t = o$.
b	constant in $X(x)$ function.
$B = B(x,t) = \rho(x,t)A(x,t)$	product of gas density and cross sectional area.
c	constant in $X(x)$ function.
$c_1 = \dfrac{\gamma p_o}{\rho_o^\gamma}$	constant.
c_p and c_v	specific heat of gas at constant pressure and constant volume respectively.
$f(t) = u(o,t)$	velocity function at $x = o$.
h	length of reed.
k	half distance between a pair of reeds at $x = o$.
$K_1(t), K_2(t)$	arbitrary functions of time arising in integrations.
m, m_o and n	constant in $f(t)$ function.
m^*	mass of reeds per unit length.
M and N	abbreviations for recurring expressions.
$p = p(x,t)$	pressure of gas.
$q = q(x,t)$	force per unit of length, forcing function of reed vibrations.
Q	abbreviation for recurring function.
t	time variable.
t_1	opening time for reed movement.
$T(t)$	time function.
$u = u(x,t)$	velocity of flow in x direction.
$v = [T(t)]^{(1-\gamma)}$	time function.
w	depth of reeds.
x	space variable.
$X(x)$	space function.
$y = \dfrac{A}{2w} = y(x,t)$	dependent space variable.

subscript o	free stream conditions.
subscript i	inflow.
subscript c	combustion chamber.
α	constant defined by equation (c.3).
β	arbitrary constant arising in integration.
$\gamma = \frac{c_p}{c_v}$	adiabatic constant.
$\eta = \left(\frac{\partial y}{\partial x}\right)_{x=o}$	tangent of reed at $x = o$.
λ_1, λ_2	abbreviations for recurring functions.
μ, σ, ε, λ	constants.
$\rho = \rho(x,t)$	density of gas.
ρ_s	stagnation density.

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